Chapter 4 delves into the core and advanced principles of machine learning and statistics, with a focus on linear regression, recursion, and regression analysis. It thoroughly examines their conceptual frameworks and practical applications in the realm of predictive modeling and data interpretation.

Linear Regression is introduced as a critical statistical approach for delineating the relationship between a dependent variable and multiple independent variables. The narrative describes how this methodology relies on computing a weighted sum of input features, augmented by a bias term, to predict outcomes. The chapter details the Ordinary Least Squares method for estimating model parameters, aiming to minimize a cost function, thus highlighting the model's capacity for prediction.

Recursion is presented as both a computational strategy and a mathematical construct where a function is recursively defined, enabling the simplification of complex problems into solvable sub-problems. The text draws an analogy to the sequential unveiling of Russian nesting dolls to illustrate recursion's efficacy in addressing problems marked by repetitive or hierarchical structures.

The discussion extends to Regression Analysis as a versatile framework for probing and modeling the interplay among several variables. This segment underscores the method's utility in predicting or elucidating a dependent variable, enriched by a discussion on the conceptual underpinnings of the model, including conditional expectation and the minimization of error terms through established cost functions.

The chapter also rigorously reviews regularization techniques like Ridge, Lasso, and Elastic Net, which are instrumental in curtailing overfitting by imposing penalties on the coefficients' magnitude. Additionally, it explores Polynomial Regression for modeling non-linear relationships, thereby extending the applicability of linear models.

In summary, Chapter 4 provides a comprehensive academic investigation into pivotal statistical and machine learning methodologies, delineating their theoretical bases, mathematical constructs devoid of explicit formulas, and practical relevance.